Anoka-Hennepin Secondary Curriculum Unit Plan

Department:	**Mathematics**	**Course:**	**AP Calculus BC**	**Unit 3 Title:**	**Parametric, Polar, and Vector**	**Grade Level(s):**	**10-12**
Assessed Trimester:	**Trimester A**	**Pacing:**	**13-19 days**	**Date Created:**	**2/2/2010**	**Last Revision Date:**	**6/19/2014**

Course Understandings: *Students will understand that:*

A. The meaning of limit represents function behavior.
B. The meaning of the derivative represents a rate of change and is a local linear approximation and should understand that derivatives can be used to solve a variety of problems.
C. The meaning of the definite integral is a limit of Riemann sums and as the net accumulation of change and will understand that you can use integrals to solve a variety of problems.
D. The relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
E. You can model a written description of a physical situation with a function, a differential equation, or an integral.
F. You can use technology to help solve problems, experiment, interpret results, and support conclusions.
H. The analysis of planar curves can be done using the calculus of parametric, vector, and polar forms.

DESIRED RESULTS (Stage 1) - WHAT WE WANT STUDENT TO KNOW AND BE ABLE TO DO?

Established Goals

Minnesota State/Local/College Board/Technology Standard(s) addressed:

- **AP: I. Functions, Graphs, and Limits**
  e. **Parametric, polar, and vector functions**
    - The analysis of planar curves includes those given in parametric form, polar form, and vector form.

- **AP: II. Derivatives**
  a. **Concept of the derivative**
    - Derivative presented graphically, numerically, and analytically
    - Derivative interpreted as an instantaneous rate of change
    - Derivative defined as the limit of the difference quotient
    - Relationship between differentiability and continuity

  a. **Derivative as a function**
    - Corresponding characteristics of graphs of f and f'
    - Relationship between the increasing and decreasing behavior of f and the sign of f'
    - The Mean Value Theorem and its geometric interpretation
    - Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

  b. **Second derivatives**
    - Corresponding characteristics of the graphs of f, f', and f''
    - Relationship between the concavity of f and the sign of f''
    - Points of inflection as places where concavity changes

  c. **Applications of derivatives**
    - Analysis of curves, including the notions of monotonicity and concavity
    - Optimization, both absolute (global) and relative (local) extrema
    - Modeling rates of change, including related rates problems
    - Use of implicit differentiation to find the derivative of an inverse function
    - Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
    - Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

  d. **Computation of derivatives**
    - Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
    - Derivative rules for sums, products, and quotients of functions Chain rule and implicit differentiation

- **AP: III. Integrals**
  a. **Interpretations and properties of definite integrals**
    - Definite integral as a limit of Riemann sums
    - Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:
    - Basic properties of definite integrals (examples include additivity and linearity)
  b. **Applications of integrals**
    - Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.
  c. **Fundamental Theorem of Calculus**
    - Use of the Fundamental Theorem to evaluate definite integrals
    - Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined
  d. **Techniques of antidifferentiation**
    - Antiderivatives following directly from derivatives of basic functions
    - Antiderivatives by substitution of variables (including change of limits for definite integrals)
  e. **Applications of antidifferentiation**
    - Finding specific antiderivatives using initial conditions, including applications to motion along a line
    - Solving separable differential equations and using them in modeling (including the study of the equation $y=ky$ and exponential growth)
  f. **Numerical approximations to definite integrals**
    - Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

Transfer

Students will be able to independently use their learning to: (product, high order reasoning)

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Meaning

Unit Understanding(s):	**Essential Question(s):**
Students will understand that: • The parametric, polar, and vector representations of planar curves • The calculus of parametric, polar, and vector forms	**Students will keep considering:** • Can I do a derivative and integral in polar form? • Can I do a derivative and integral in parametric form? • Can I do a derivative and integral in vector form? • Are certain planar curves better represented in one form vs. another?

Acquisition

Knowledge - Students will: • Displacement vs. total distance traveled • Polar vs. rectangular coordinate plane • Definition of a vector • Conversion formulas (polar-rectangular) **Reasoning - Students will:** • Analyze position, velocity, and acceleration of a particle using parametric and polar coordinates • Analyze graphs to determine how to set up integrals to calculate area • Analyze graphs to determine how to set up integrals to calculate arc length • Justify the second derivative rule for parametric equations using the chain rule • Interpret results in the context of position, velocity, and acceleration • Determine which component of the parametric equation to use to determine extrema	**Skills - Students will:** • BC3-1: Graphing the path of a particle in parametric or polar • BC3-2: Calculate the area bounded by polar curves • BC3-3: Calculate the length of a polar curve/parametric curve • BC3-4: Calculate first and second derivative of parametric curves • BC3-5: Convert between polar, parametric, and rectangular forms • BC3-6: Use vectors to model position, velocity, and acceleration • BC3-7: Calculate derivatives and integrals of vector valued functions • BC3-8: Using polar and parametric equations to determine extrema

Common Misunderstandings	Essential new vocabulary
• Students have trouble finding the limits of integration while calculating area bounded by a polar curve • Students have trouble finding equations of tangent lines • Students have trouble converting from polar to rectangular • Second derivative in parametric equations needs to divide by dx/dt • Students often use $dr/d\theta$ for slope of a polar	• Magnitude and direction of a vector • Speed • Standard Form of a vector • Unit vectors • Vertical and horizontal • Vectors